CSE311 Microwave Engineering

LEC (06)
Transmission Lines_ Part II

Assoc. Prof. Dr. Moataz Elsherbini

motaz.ali@feng.bu.edu.eg

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Example 3.1

The parameters of a certain transmission line operating at ω = 6x10^8 rad/sec are: L = 0.4 μH/m, C = 40 pF/m, G = 80 mS/m and R = 20 Ω/m. If the voltage wave travels a length of the line, l = 20 m down the line, find:

a) The complex propagation constant γ, the attenuation constant α and phase constant β.

b) The wave parameters: the phase velocity V_p, the wavelength λ and the characteristic impedance Z_o.

c) The ratio of the wave amplitude (at z = 20m) to the initial wave amplitude (at z =0).

d) The phase shift at the end of the line (at z = l = 20m) from the initial (at z = 0).

Solution:

a) From Eqn. (3.9)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{[20 + j(6x10^{8})(0.4x10^{-6})][80x10^{-3} + j(6x10^{8})(40x10^{-12})]}$$

$$= (2.8 + j3.5) \;\; = \alpha + j\beta$$

3.2 The Transmission Line Equations (Continued)

3.2.3 Transmission Line wave parameters(Continued)

Therefore α = 2.8 Np/m and β = 3.5 rad/m

b) From Eqn. (3.16) $V_p = \dfrac{\omega}{\beta} = \dfrac{6x10^8}{3.5} = 1.714x10^8 \ m/\sec$

From Eqn. (3.17) $\lambda = \dfrac{2\pi}{\beta} = 1.8 \ m$

From Eqn. (3.13) $Zo = \sqrt{\dfrac{R + j\omega L}{G + j\omega c}} = \sqrt{\dfrac{20 + j2.4x10^2}{80x10^{-3} + j2.4x10^{-2}}} = (44 + j30) \ \Omega$

c) From Eqn.(3.10) for the forward wave

$$V(z) = V_o^+ e^{-\gamma z} = V_o^+ e^{-\alpha z} e^{-j\beta z}$$

Therefore $\dfrac{V_o^+ e^{-\alpha z}}{V_o^+} = e^{-\alpha z} = e^{-(2.8)(20)} = 6.9x10^{-13}$

d) The phase shift is β*l* = (3.5)(20)x 180/π = 4010º = 50º

3.2 The Transmission Line Equations (Continued)

3.2.4 The Lossless Transmission Line

- The above solution was for general transmission line, including loss effects. The propagation constant and characteristic impedance are complex.
- In many practical cases, the loss of the line is very small and so can be neglected.
- Setting R = G = 0 in (3.9) gives the propagation constant as:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC} \qquad (3.18)$$

$$\alpha = 0 \;\; , \;\; \beta = \omega\sqrt{LC} \qquad (3.19)$$

- As expected for the lossless case, the attenuation constant α is zero, which indicates no change in the amplitude of the wave.
- The characteristic impedance (3.13) reduces to:

$$Z_o = \sqrt{L/C} = |Z_o| e^{j\theta} \;\; \Rightarrow |Z_o| = \sqrt{L/C} \;\; and \;\; \theta = 0^o \qquad (3.20)$$

Therefore, the wave parameters are:

1. The phase velocity V_p is: $v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}}$ (3.21)
2. The wavelength λ is: $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{\omega\sqrt{LC}}$ (3.22)
3. The characteristic impedance Z_o is (3.13): $Z_o = \sqrt{L/C}$ (3.23)

3.2 The Transmission Line Equations (Continued)

3.2.4 The Lossless Transmission Line (Continued)

- For lossless transmission line, the general solution for voltage and current can be written as:

$$V_s(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z} \qquad (3.24)$$

and

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z} \qquad (3.25)$$

- The real instantaneous voltage and current can be obtained by multiplying (3.24) and (2.25) by $e^{j\omega t}$, $\theta = 0^o$ and then taking the real part we get:

$$v(z,t) = V_o^+ \cos(\omega t - \beta z) + V_o^- \cos(\omega t + \beta z) \qquad (3.26)$$

$$i(z,t) = \frac{V_o^+}{z_o} \cos(\omega t - \beta z) - \frac{V_o^-}{z_o} \cos(\omega t + \beta z) \qquad (3.27)$$

Example 3.2:

A lossless transmission line of length, l = 80 cm and operates at a frequency, f = 600 MHz. The line parameters are L = 0.25 µH/m and C = 100pF/m, find:

a)The propagation constant γ, the attenuation constant α and phase constant β.

b)The wave parameters: the phase velocity V_p, the wavelength λ, and the characteristic impedance z_o.

c)The voltage amplitude and the phase shift at the end of the line, l.

3.2 The Transmission Line Equations (Continued)

3.2.4 The Lossless Transmission Line (Continued)

Example 3.2: Solution

a) From (3.18) the propagation constant γ is

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega c)} = \sqrt{(j\omega L)(j\omega c)}$$

$$= j\omega\sqrt{LC} = j(2\pi)(600x10^{6})\sqrt{(0.25)(10^{-6}x100x10^{-12})} = j18.85$$

Therefore, α = 0 and β = 18.85 rad/m

b) V_p = ω/β = 2πx6x10^8/ 18.85 = 2x10^8 m/sec (C = 3x10^8 m/sec)

λ = 2π/β = 0.333 m (λo = c/f = 0.5 m)

$$z_o = \sqrt{\frac{L}{c}} = \sqrt{\frac{0.25x10^{-6}}{100x10^{-12}}} = 50\angle 0^{o}\ \Omega = 50\ \Omega$$

c) Since α = 0, the voltage amplitude of the wave does not change at any point. The phase shift at the end of the line is βl = 18.85 x 0.8 x (180/π) = 864.02^o = 144^o

3.3 Power Transmission over the Transmission Line

- Having found the sinusoidal voltage and current in a lossy transmission line, (3.15), we next evaluate the power transmitted over a specified distance as a function of voltage and current amplitudes.
- The instantaneous power, given simply as the product of the real voltage and current. Consider the forward propagating terms in (3.15), where again, the amplitude, $V_o^+ = |V_o|$, is taken to be real.
- The current waveform will be similar, but will generally be shifted in phase.
- Both current and voltage attenuate according to the factor $e^{-\alpha z}$. The instantaneous power therefore becomes:

$$P(z,t) = v(z,t)I(z,t) = |V_o||I_o|e^{-2\alpha z}\cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta) \quad (3.28)$$

- Usually, the time-average power, $P_{av}(z)$, is of interest. We find this through:

$$P_{av}(z) = \frac{1}{T}\int_o^T |V_o||I_o|e^{-2\alpha z}\cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta)dt \quad (3.29)$$

- Where T = 1/f = 2π/ω is the period for one oscillation cycle.

3.3 Power Transmission over the Transmission Line (cont.)

- Using a trigonometric identity, the product of cosines in the integral can be written as the sum of individual cosines at the sum and difference frequencies:

$$P_{av}(z) = \frac{1}{T}\int_o^T \frac{1}{2}|V_o||I_o|e^{-2\alpha z}[\cos(2\omega t - 2\beta z - \theta) + \cos(\theta)]dt \qquad (3.30)$$

- The first cosine term integrates to zero, leaving the cos θ term. The remaining integral easily evaluates as

$$P_{av}(z) = \frac{1}{2}|V_o||I_o|e^{-2\alpha z}\cos\theta = \frac{1}{2}\frac{|V_o|^2}{|Z_o|}e^{-2\alpha z}\cos\theta \quad [W] \qquad (3.31)$$

- The same result can be obtained directly from the phasor voltage and current. We begin with these, expressed as

$$V_s(z) = V_o e^{-\alpha z} e^{-j\beta z} \qquad (3.32)$$

and

$$I_s(z) = I_o e^{-\alpha z} e^{-j\beta z} = \frac{V_o}{Z_o} e^{-\alpha z} e^{-j\beta z} \qquad (3.33)$$

- Where $Z_o = |Z_o|e^{j\theta}$. We now note that time-average power as expressed in (3.31) can be obtained from the phasor forms through:

$$P_{av}(z) = \frac{1}{2}\mathrm{Re}[VI^*] = \frac{1}{2}\mathrm{Re}[V_s I_s^*] \qquad (3.34)$$

3.3 Power Transmission over the Transmission Line (cont.)

- Where again, the asterisk (*) denotes the complex conjugate (applied in this case to the current phasor only). Using (3.32) and (3.33) in (3.34) it is found that

$$P_{av}(z) = \frac{1}{2}\mathrm{Re}\left[V_o e^{-\alpha z} e^{-j\beta z} \frac{V_0^*}{|Z_o| e^{j\theta}} e^{-\alpha z} e^{+j\beta z}\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{V_o V_0^*}{|Z_o|} e^{-2\alpha z} e^{-j\theta}\right] = \frac{1}{2}\frac{|V_0|^2}{Z_o} e^{-2\alpha z}\cos\theta \qquad (3.35)$$

- Which we note is identical to the time-integrated result in (3.31). Eqn.(3.34) applies to any single-frequency wave.
- An important result of the preceding exercise is the power attenuates as $e^{-2\alpha z}$, or

$$P_{av}(z) = P_{av}(0) e^{-2\alpha z} \qquad (3.36)$$

- Power drops at twice the exponential rate with distance as either voltage or current. A convenient measure of power loss is in decibel units. This is based on expressing the power decrease as a power of 10. specifically, we write

$$\frac{P_{av}(z)}{P_{av}(0)} = e^{-2\alpha z} = 10^{-k\alpha z} \qquad (3.37)$$

- Where the constant, k, is to be determined. Setting $\alpha z = 1$, we find that $k = 0.869$

3.3 Power Transmission over the Transmission Line (cont.)

- By definition, the power loss in decibels (dB) is:

$$\text{Power loss (dB)} = 10\log_{10}\left(\frac{P_{av}(0)}{P_{av}(z)}\right) = 8.69\alpha z = 20\log_{10}\left(\frac{|V(0)|}{|V(z)|}\right) \qquad (3.38)$$

- Where $|V(z)| = |V_o(0)|e^{-\alpha z}$

Example 3.3 A length of transmission line, l = 20m is known to produce a 2.0 dB drop in power from end to end. For this line calculate the following:
(a) The fraction of the input power that reaches the output.
(b) The fraction of the input power reaches the midpoint of the line.

Solution

(a) The power fraction will be $10\log\left(\frac{P_{av}(20)}{P_{av}(0)}\right) = -2\ dB \Rightarrow P_{av}(20\ m) = 0.63\ P_{av}(0)$

(b) 2 dB in ℓ = 20 m implies a loss rating of 0.1 dB/m. So, over a ℓ/2 = 10 meter span, the loss is 1.0 dB. This represents the power fraction, $10^{-0.1}$ = 0.79.

Fig.3.5 Section of transmission line for attenuation calculation.

3.4 Wave Reflection at Discontinuities of TL

- A reflected wave originates from the necessity to satisfy all voltage and current boundary condition at the end of transmission lines and at locations at which two dissimilar lines are connected to each other (discontinuities).
- The consequences of the reflected waves are usually less than desirable, in that some of the power that was intended to be transmitted to the load reflects and propagates back to the source.
- Fig. 3.6 shows a lossless line terminated in an arbitrary load impedance Z_L. This problem will illustrate wave reflection on the transmission line, a fundamental property of distributed systems.
- For a lossless transmission line, $\alpha = 0$, and the characteristic impedance is Z_o. We assign coordinates such that the load location is, $z = 0$, therefore the line occupies the region $z < 0$.

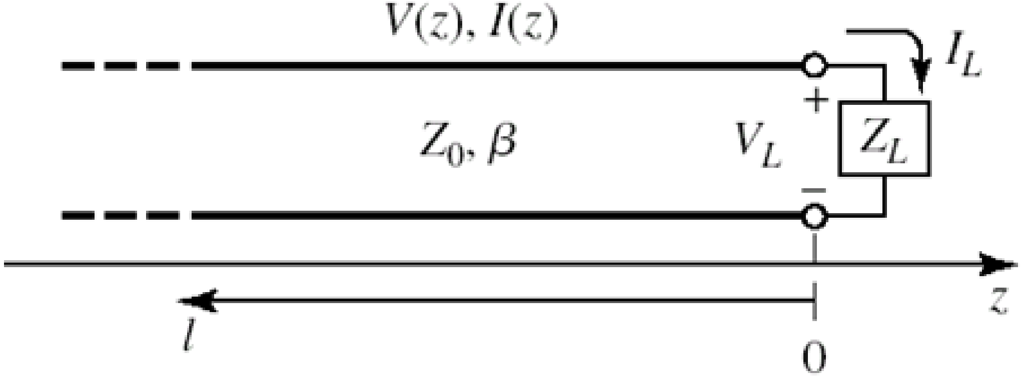

Fig. 3.6 A transmission line terminated in a load impedance Z_L.

3.4 Wave Reflection at Discontinuities (Continued)

- Assume that an incident wave is generated from a source at Z < 0, to be incident on the load, and is expressed in phasor for all z as:

$$V_{is} = V_o^{+} e^{-j\beta z} \qquad (3.41)$$

- The ratio of voltage to the current for such traveling wave is the characteristic impedance (Z_o). But when the line is terminated in an arbitrary load ($Z_L \neq Z_o$), the ratio of voltage to current at the load must Z_L. Thus, a reflected wave must be excited with appropriate amplitude to satisfy this condition.
- The total voltage on the line can be written as the sum of incident and reflected waves, also in phasor we have:

$$V_s(z) = V_o^{+} e^{-j\beta z} + V_o^{-} e^{j\beta z} \qquad (3.42)$$

- Similarly, the total current on the line as described by (3.25) is:

$$I_s(z) = \frac{V_o^{+}}{Z_o} e^{-j\beta z} - \frac{V_o^{-}}{Z_o} e^{j\beta z} \qquad (3.43)$$

- Where $Z_o = |Z_o|e^{j\theta}$. The total voltage and current at the load are related by the load impedance, Z_L, at z = 0, so we must have:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^{+} + V_o^{-}}{V_o^{+} - V_o^{-}} Z_o$$

3.4 Wave Reflection at Discontinuities (Continued)

- Solving for V_o^- gives:

$$V_0^- = \frac{Z_L - Z_o}{Z_L + Z_o} V_0^+$$

- The amplitude of the reflected voltage wave normalized to the amplitude of incident voltage wave is defined as the reflection coefficient, Γ and is given by:

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma| e^{j\varphi} \qquad (3.44)$$

where $|\Gamma|$ is the magnitude and φ is the phase of the reflection coefficient.

- The total voltage and current waves on the line can be written as:

$$V_s(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}] \qquad (3.45)$$

and

$$I_s(z) = \frac{V_o^+}{Z_o}[e^{-j\beta z} - \Gamma e^{j\beta z}] \qquad (3.46)$$

- From (3.45) and (3.46), it is seen that the voltage and current on the line consist of a superposition of incident and reflected wave; such waves are called standing wave that will be studied in more details in the next section.
- When $\Gamma = 0$, there is no reflected wave. To obtain $\Gamma = 0$, the load impedance Z_L must be equal to the characteristic impedance Z_o of the transmission line as seen from (3.44). Such a load is then said to be matched to the line.

3.4 Wave Reflection at Discontinuities (Continued)

Example 3.4:

The characteristic impedance of a lossless transmission line is Z_o = 72 Ω, if it is working at frequency of (f = 80 MHz), L = 0.5 µH/m and the line is terminated with load Z_L = 60 Ω, find:

a) The shunt capacitance per m, C, the phase velocity, Vp and the phase constant, β.

b) The reflection coefficient, Γ.

Solution:

a) From (3.23) Z_o is: $Z_o = \sqrt{L/C}$

then $$C = \frac{L}{Z_o^2} = \frac{5x10^{-7}}{(72)^2} = 9.6x10^{-11}\,F/m = 96\ \ PF/m$$

From (3.22) Vp is: $$V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5x10^{-7})(9.6x10^{-11})}} = 1.44x10^8\ m/\sec$$

From (3.19) β is: $$\beta = \omega\sqrt{LC} = \frac{2\pi x 80x10^6}{1.44x10^8} = 3.5\ rad/\sec$$

b) From Eqn. (3.44) Γ is: $$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{60-72}{60+72} = -0.09 = 0.09\angle 180^o$$

3.4 Wave Reflection at Discontinuities (Continued)

- Now consider the time average power flow along the line at a point z (3.35), using (3.45) and (3.46) we get:

$$V_s(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}]$$

$$P_{av} = \frac{1}{2}\text{Re}[V(z)I^*(z)] = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}\text{Re}[1 - \Gamma^* e^{-j2\beta z} + \Gamma e^{j2\beta z} - |\Gamma^2|]$$

- The middle two terms in the brackets are A – A* = 2j Im (A) and so are pure imaginary. This simplifies the result to:

$$I_s(z) = \frac{V_o^+}{Z_o}[e^{-j\beta z} - \Gamma e^{j\beta z}]$$

$$P_{av} = \frac{1}{2}\text{Re}[V(z)I^*(z)] = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}(1 - |\Gamma^2|) \qquad (3.47)$$

- Eqn. (3.47) shows that the average power flow is constant at any point on the line, and that the total power delivered to the load (P_{av}) is equal to the incident power $P_i=[V_o^{+2}/(2Z_o)]$ minus the reflected power $P_r=[V_o^{+2}\,|\Gamma|^2/(2Z_o)]$.
- If Γ = 0, the maximum power is delivered to the load.
- If Γ = 1, no power is delivered.
- The above discussion assumes that the generator is matched, so that there is no reflection of the reflected wave from z < 0.
- When the load is mismatched, not all of the available power from the generator is delivered to the load. This loss is called Return Loss (RL) and is defined as:

$$RL = 20\log|\Gamma| \quad dB \qquad (3.48)$$

- A matched load (Γ = 0) has a return loss of - ∞ dB (no reflected power).
- A total reflection (|Γ| = 1) has return loss of 0 dB which means that all incident power is reflected.

3.4 Wave Reflection at Discontinuities (Continued)

Example 3.5

A 50 Ω lossless transmission line is terminated by a load impedance Z_L= 50 - j75 [Ω]. If the incident power is 100 mW, find the power dissipated by the load.

Solution:

The reflection coefficient Γ is:

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{50 - j75 - 50}{50 - j75 + 50} = 0.36 - j0.48 = 0.6e^{-j0.93}$$

- The power dissipated by the load

$$P_{avL} = P_i(1 - \left|\Gamma^2\right|) = 100[1 - (0.6)^2] = 64\ mw$$

3.5 Voltage Standing Wave Ratio (VSWR)

$$V_s(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}]$$

- From Eqn. (3.45), the voltage on the line is the superposition of the incident wave $V_o^+ e^{-j\beta z}$ and the reflected wave $V_o^+ \Gamma e^{j\beta z}$.
- If the load is matched, Γ = 0, the magnitude of the voltage on the line is $|V(z)| = |V_0^+|$, which is constant. Such line is sometimes said to be "flat".
- When the load is mismatched, the presence of a reflected wave leads to a standing waves, where the magnitude of the voltage on the line is not constant. Thus from (3.45) we have: